Earthquake Analysis of MDOF System Using Linear Fluid Viscous Damper

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Abstract —
With advances in technology, it appears that the approach to the design of earthquake resisting structures takes a new direction, which allows engineers to design structures for a desired level of seismic protection. Designing structures to behave elastically or near the elastic range during strong ground motions is not economical, and in many cases is not feasible. Therefore enabling the structure to dissipate energy by means of mechanical devices appears very attractive. A rich variety of energy dissipation devices for passive control may be found in viscous dampers seem more appropriate in the case of rehabilitation. The main advantage of viscous dampers is that the forces they produce are out of phase with the column’s forces due to displacements, and therefore, will not usually require column and foundation strengthening.

OVERVIEW ON DAMPING
Consider a building roof that has been pulled over horizontally a small amount and then released from rest. The roof will vibrate back and forth with amplitude that diminishes with time. Historically, structural engineers assumed that the reduction in motion was associated with the presence of viscous damping. The scientific quantification of the value of viscous damping in buildings has been the subject of research for over 50 years. This type of damping is called natural damping. This natural damping represents the energy dissipated by structural materials (e.g. concrete, steel and masonry) and non-structural elements (e.g. partitions, cladding) as the building moves with time.

The level of damping (natural damping) in a conventional elastic structure is very low, and hence the amount of energy dissipated during transient disturbances is also very low. During strong motions, such as earthquakes, conventional structures usually deform well beyond their elastic limits, and eventually fail or collapse. Therefore, most of the energy dissipated is absorbed by the structure itself through localized damage as it fails. The concept of supplemental dampers added to a structure assumes that much of the energy input to the structure from a transient will be absorbed, not by the structure itself, but rather by supplemental damping elements (manufactured damping). An idealized damper would be of a form such that the force being produced by the damper is of such a magnitude and function that the damper forces do not increase overall stress in the structure. Properly implemented, an ideal damper should be able to simultaneously reduce both stress and deflection in the structure. In the late 1960s, the structural engineers in Los Angeles and California installed instruments called accelerometers in buildings to measure building response during an earthquake. Usually these accelerometers are attached slab or a building floor. The recorded motion of the building can then be used to estimate the magnitude of the natural damping in the building. Building responses to earthquake and strong winds have been used to obtain these estimates of natural damping.

FLUID VISCOUS DAMPERS
In the 1990s, there was a virtual explosion in the use of new high-technology structural element that dissipates energy using a viscous damper. This type of damping is called manufactured viscous damping or simply manufactured damping because the damper is manufactured in plant under exacting quality control standards. These manufactured viscous dampers (or manufactured dampers) are then to sent the building site to be installed. Manufactured dampers are high-technology structural elements that have been used by the military and by the automobile and ship industries.
Fig.1. Fluid viscous damper (Taylor’s device)

Fig.1 shows the schematic of a typical viscous damper. The scientific/engineering design varies among manufactures of viscous damper, but the basic performance from a structural engineering perspective is the same for all viscous dampers. The viscous damper is attached to the building structure either as a part of diagonal brace between the floors of the building or at the top of such a brace between the bracing unit and the floor. As the floor move, their relative velocity imparts a differential velocity to the ends of the viscous damper and therefore the damper exerts a viscous force to the floor.

LINEAR FLUID VISCOUS DAMPERS

The two ends of viscous damper experience different displacement, velocity and acceleration because of one end is attached to one building floor and other end to different floor. This difference in the motion results in the viscous damper producing a force and a source of energy dissipation. The force induced to the structure by the viscous damper at each of the attachment point of the viscous damper can be expressed as

\[ F_{md} = c_{md} (u\dot{u})^\eta \]

Where \( F_{md} \) is the damping force from the manufactured viscous damper, \( c_{md} \) is the manufactured viscous damper damping coefficient, \( u \) is the relative velocity between the ends of the viscous damper and \( \eta \) is a power law coefficient.

We are interested here only for linear systems. The first generation of manufactured viscous damper used a power law coefficient equal to one(1). structural engineers selected this value for \( \eta = 1 \) for design because for \( \eta = 1 \) the manufactured damping force, is linear function of velocity.

Fig.9. Shows the damping force is a linear function of velocity if the power low coefficient \( \eta = 1 \). It means that the velocity of system or structure linearly varies with the damping force \( F_d \) here \( F_d \) is the total force of damping. Therefore, for a single degree of freedom system the total damping force is;

\[ F_d = F_{nd} + F_{md} \]

Where \( F_d \) is the total damping force, \( F_{nd} \) is the force due to natural damping force, and \( F_{md} \) is the manufactured damping force. Where both the natural and manufactured damping forces are a linear function of velocity, it follows that,

\[ F_{nd} = c_{nd} \dot{u} (t) \]
\[ F_{md} = c_{md} u (t). \]

EQUATION OF MOTION AND PROBLEM STATEMENT

PROBLEM STATEMENT

One of the most important application of the theory of structural dynamics is in analyzing the response of structure to ground shaking caused by an earthquake. We are interested in the response of linear systems.

Analytical solution of the equation of motion for a system subjected to any applied force \( P(t) \), or ground acceleration \( U_g(t) \) is not possible, because here the force or ground motion is the function of \( t \), which is arbitrarily varies with time. The analytical solution if this type of problem is very rigorous. Such a problem can be tackled by a numerical time stepping methods. Some methods are listed below

1) Newark’s method  2) Wilson \( \theta \)-method

For this paper we accept the Newark’s method for integrate the equation of motion. This method is
developed by N.M.Newmark in 1959. Parenthetically, the Newark’s method is the time stepping method where we can define the discrete time instants Δ(t), and introduce the initial displacement(u), velocity(û), and acceleration(û̈) is equal to zero. After that we can find out the response increasing by

Δ(t) and (u_{i+1}), (û_{i+1}), (û̈_{i+1}) at Δ(t_{i+1}).

EQUATION OF MOTION

The dynamic equation of motion for an SDOF system subjected to an earthquake ground motion is

\[ m \ddot{u}(t) + c_{nd} \dot{u}(t) + c_{md} \dot{u}(t) + k u(t) = -m \ddot{u}(t). \]

After dividing both sides of above equation by the mass (m), it becomes

\[ \ddot{u}(t) + 2ζ \omega_n \dot{u}(t) + \omega_n^2 u(t) = - \ddot{u}(t). \]

where \( \omega_n = \sqrt{k/m} \), \( \zeta_{nd} \) is the critical damping ratio from natural damping, and \( \zeta_{md} \) is the critical damping ratio from manufactured damping. These damping ratios are defined using equation

\[ 2 \zeta_{md} \omega_n = c_{md}/m \]
\[ 2 \zeta_{nd} \omega_n = c_{nd}/m \]

The above equation can be written as

\[ \ddot{u}(t) + 2ζ \omega_n \dot{u}(t) + \omega_n^2 u(t) = \ddot{u}(t). \]

Where \( \zeta \) represents the total critical damping ratio of the structure and it is

\[ \zeta = \zeta_{md} + \zeta_{nd} \]

The modified equation is same as the above original equation of SDOF, with the critical damping ratio now being the summation of the natural and manufactured critical damping ratios. Therefore for the case where \( \eta = 1 \), the structural system is the same as per the above system. It should be noted that, the introduction of manufactured damping to the structure does not introduce any new solution complexity. The earthquake acceleration in the above equation is very irregular and cannot be described by a closed form mathematical function; therefore a numerical solution is necessary. The numerical methods mentioned in above e.g. Newmark’s method or Wilson θ-method can be used to solve the equation of motion.

SYSTEM PARAMETERS

As indicated by above equation, the response of SDOF system with linear fluid viscous dampers is controlled by four parameters.

1. Damper linearity parameter \( \eta \) which controls the shape damper force in fig. 2.
2. Supplement damping ratio \( \zeta_{md} \), which represents the energy dissipation capacity of the fluid viscous dampers.
3. Natural vibration period of the system \( T_n = 2\pi/\omega_n \) and ,
4. Damping ratio \( \zeta \) which represents the inherent energy dissipation capacity of the system.

The inherent damping of the SDOF system was fixed at \( \zeta = 5\% \) and its natural vibration period \( T_n \) was varied from 0.05 to 5sec

INCREASE IN LINEAR MODAL DAMPING

We mentioned above there is no hard and fast rule for introducing a manufactured damping in the model. We can easily understand by an example…..

→ e.g., consider an SDOF system with only natural damping. If the value of natural damping is 5% of critical, then

\[ \zeta_{nd} = 0.05\% \]

let the mass of the structure be \( m = 1kg \) and the period of vibration be \( T_n = 1sec \), then

\[ \omega_n = 2\pi/T_n \]
\[ \omega_n = 2\pi \]
\[ c_{nd} = 2\zeta_{nd}\omega_n m = 0.251 \text{ N-s/m} \]

Assume that the structural engineer wants to increase the damping of the structure to 15% of critical, that is

\[ \zeta = 0.15 \]

then it follows that the manufactured damping should be

\[ \zeta_{md} = \zeta - \zeta_{nd} = 0.15 - 0.05 = 0.10 \]

and therefore the manufactured viscous damping coefficient must be equal to

\[ c_{md} = 2\zeta_{md}\omega_n m = 1.2566 \text{ N-s/m} \]

The selection of the damping coefficient \( c_{md} \) is a function of the desired damping ratio (0.10) and also the natural frequency of vibration and the mass of the structure.

When manufactured dampers are ordered, one critical design variable that must be specified is the maximum damping force. Usually the structural engineer will know from structural engineering calculations the maximum relative velocity that the design earthquake will induce on the two ends of viscous damper. Therefore, if the maximum velocity is \( \dot{u}_{max} \) then the maximum force required for the manufactured damper is

\[ F_{md} = c_{md} \dot{u}_{max} \]

EARTHQUAKE RESPONSE

Here we considered the one ground motion which is recorded at site in El Centro, California, during the imperial Valley, California earthquake of May 18,1940.it should be noted that the ground acceleration with respective to time is highly irregular. How much irregular, it is not a problem.
The above figure shows the ground motion recorded at site El-Centro ground motion, the ground acceleration $\ddot{u}_g(t)$ defined at every 0.02 sec.

**RESPONSE QUANTITIES**

1. Peak deformation ($u_{max}$) to which the peak lateral force $F=Ku_{max}$ and internal forces in the structure are related.
2. Peak damper force $F_d$.
3. Peak acceleration ($\ddot{u}$) of the mass.
4. Peak value of relative velocity $\dot{u}$ which is necessary to compute the exact forces in a linear fluid viscous damper and to verify the accuracy of its approximate value.

**NUMERICAL EXAMPLES**

→ Consider the SDOF system subjected to NS component of El Centro earthquake 1940(California), with natural period $T_n=0.5$ sec, and natural damping coefficient $\zeta=5\%$, $m=1$ kg.

→ Here we already know the variation of El-Centro ground acceleration is very irregular in the above fig.4. the graph representing the variation of ground acceleration ($\ddot{u}_g(t)$) with respective time. Hence the analytical solution of equation of motion is not possible or which required much more effort. So, we have to use the any numerical method mention in above literature to find out the response of structure, once the basic response displacement($u$), velocity($\dot{u}$), acceleration($\ddot{u}$) is find out we are able to calculate the internal forces of structure easily by simple mathematical formulations.

**RESPONSE ONLY WITH NATURAL DAMPING**

"Fig.3. Deformation response of system subjected to El-Centro ground motion ($T_n=0.5$ sec, $\zeta=5\%$)"
Fig. 4. Velocity response of system subjected to El-Centro ground motion ($T_n=0.5$ sec, $\zeta_n=5\%$)

Fig. 5. Acceleration response of system subjected to El-Centro ground motion ($T_n=0.5$ sec, $\zeta_n=5\%$)

The above three figures show the response of structure with natural damping only, means ($\zeta_n=5\%$)

RESPONSE WITH NATURAL AND MANUFACTURED DAMPING

The introduction of manufactured damping to the structure does not introduce any new solution complexity. When manufactured damping is introduced the structural response will be decreased, here we will increase the manufactured damping with $\zeta_{md}=10\%$, then after the response of structure will be as shown in figures 6, 7, 8.

Fig. 6. Deformation response of system subjected to El-Centro ground motion ($T_n=0.5$ sec, $\zeta_n=5\%$ & $\zeta_{md}=10\%$)
The power law coefficient is $\eta=1$, because of this the system is linear. We can also find the graph between velocity and damping force is linear. The damping force is:

$$F_d = c(\dot{u})^\eta$$

![Damping force versus velocity](image)

**Fig. 9. Damping force versus velocity**

![Damping force versus deformation](image)

**Fig. 10. Damping force versus deformation**
We will summarize the response in parenthetically in the table.

<table>
<thead>
<tr>
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<th>20% (manufactured)</th>
<th>30% (manufactured)</th>
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<tr>
<td></td>
<td>u</td>
<td>u</td>
</tr>
<tr>
<td></td>
<td>0.0225</td>
<td>0.3536</td>
</tr>
</tbody>
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Observe in the above table when the manufactured damping is increase in the system the response of system goes on decreasing. Here we calculate the response of structure whose natural time period $T_n=0.5$ sec, but the natural time of variation of structure will be changed for the different types of structures. The natural vibration period of structure generally defined in 0.05 sec to 5 seconds. This wide range of natural time period will be defined in the one graph for design of new structures or retrofitted to existing structures.

**RESPONSE SPECTRUM**

The central concept in earthquake engineering, the response spectrum provides a convenient means to summarize the peak responses of all possible linear SDOF system to a particular component of ground motion. It also provides a practical approach to apply the knowledge of structural dynamics to the design of structures and development of lateral force requirement in building codes. A plot of the peak value of the system, or a related parameters such as circular frequencies $\omega_n$ or cyclic frequency $f_n$, is called the response spectrum for that quantity. Each such plot is for SDOF system having a fixed damping ratio $\zeta$ are included to cover the range of damping values encountered in actual structures. Weather the peak responses are plotted against $f_n$ or $T_n$ is a matter of personal preference. We have chosen the latter because engineers prefer to use natural period rather than natural frequency because the period of vibration is a more familiar concept and one that is intuitively appealing. The deformation response is a plot of $u_{\text{max}}$ against $T_n$ for fixed $\zeta$. A similar plot for $\dot{u}_{\text{max}}$ is the maximum velocity response spectrum, and for $\ddot{u}$ is the acceleration response spectrum.

**MDOF SYSTEM**

The response of multi- degree of freedom system is obtained using the same method discussed above. Typically the system damping matrix is not proportional. Therefore the system response must be obtained using a numerical solution of the second order differential equation with the numerical methods mentioned above.
Fig. 12. Velocity response spectra (values represent the critical damping ratio $\zeta$).

Fig. 13. Acceleration response spectra (values represent the critical damping ratio $\zeta$).
CONCLUSION

This investigation for earthquake response of single-degree of freedom (SDOF) system with linear fluid viscous damper has led to the following conclusions.
1. Supplemental damping reduces structural responses, with greater reduction achieved by increasing the damping.
2. Supplemental damping is more effective in reducing the structural deformation, and hence internal forces, compared to relative velocity and or total acceleration, the latter two responses are reduced to a similar degree.
3. The design values of structural deformation and forces for a system (period $T_n$ and inherent damping $\zeta$) with linear fluid viscous damper $s$ can be estimated directly from the design spectrum for the period $T_n$ and total damping.
4. The peak value of earthquake induced force in a linear fluid viscous damper can be estimated with reasonable accuracy from the peak damper force in the corresponding linear systems, its peak deformation and relative velocity.

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